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Numerical investigation of light-wave localization in optical Fibonacci superlattices with symmetric internal structure

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Abstract. We study numerically the optical transmission of one-dimensional binary quasiperiodic dielectric multilayers, which are arranged in Fibonacci sequences along two opposite directions and possess a mirror symmetry. We find that the transmission coefficient is unity for all sequences studied at the central wavelength $\lambda = \lambda_0$, where $\lambda_0 = 4n_{A\,(B)}d_{A\,(B)}$, with $n_{A\,(B)}$ and $d_{A\,(B)}$ being the index of refraction and thickness of two kinds of layer, respectively. As the number of layers in the sequence increases, more and more perfect transmission peaks appear. We observe a scaling of the transmission spectra with increasing sequence length. These phenomena will find applications in fabrication of multiwavelength narrow-band optical filters.

1. Introduction

The study of electron localization in random systems has been an active field of research. From scaling theory [1, 2], it is known that all of the states are localized in one- and two-dimensional systems. However, the experimental observation of this phenomenon has been hampered by some possible interactions (electron–electron and electron–phonon) in real materials. It was later realized that the localization can be expected for any wave phenomenon, such as optical waves [3]. Since then, there has been increasing interest in studies of the localization of electromagnetic waves in photonic band-gap (PBG) materials [4–8]. This interest is partly due to the fact that the experimental study of the localization of light waves in an optical medium can be carried out at room temperature instead of the very low temperatures required in the study of electron localization. Besides, because of the nonexistence of the electron–electron and electron–phonon interactions in the optical case, the study of electromagnetic waves provides a possibility of testing for Anderson localization. Furthermore, the unusual property of the control of the propagation of light in PBG has potential applications in many optical devices [7–9].

In the past few years, the studies of PBG have been extended to photonic quasiperiodic structures [10–16]. As is well known, the electronic and phonon spectra of the one-dimensional Fibonacci chain or Fibonacci multilayer are described by a Cantor set with zero Lebesgue measure [17, 18]. For exhibiting the physical effects of quasiperiodic order, Kohmoto *et al* [10] proposed the photonic Fibonacci lattice and predicted that the transmission spectrum would have a multifractal structure. Recently, the experimental realization of optical Fibonacci dielectric multilayers has been reported [12–14]. The scaling of the transmission coefficient

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with increasing Fibonacci sequence length has been experimentally observed [12, 13]. These experimental results are in good agreement with theory [10].

On the other hand, Dunlap *et al* [19] pointed out that, in a one-dimensional random-dimer model (RDM), a small proportion of extended states can be found. The basic reason for the appearance of extended states in this system has been traced to the existence of symmetric internal structure [20]. Xiong [21] studied the transmission of electrons through a random array of a number of identical multibarriers and found that there exist electrons with special energies which are completely unscattered. Along these lines, it may be a valid question to ask how the symmetric internal structure in quasiperiodic optical multilayers influences the transmission property.

This paper is organized as follows. In section 2, we present the model and the methods that we are concerned with. In section 3, we study numerically the optical transmission properties. Finally, we give a brief summary and discussion in section 4.

2. Models and formalism

Let us consider a multilayer in which two types of layer A and B are arranged in a binary Fibonacci sequence. Then, we have a simple binary symmetric Fibonacci sequence (SFS) which is constructed as $S_j = \{G_j, F_j\}$, where F_j and G_j are Fibonacci sequences, which obey the recursion relations $F_j = \{F_{j-2}, F_{j-1}\}$ and $G_j = \{G_{j-1}, G_{j-2}\}$, for $j \geqslant 1$, with $F_0 = G_0 = \{\mathrm{B}\}$ and $F_1 = G_1 = \{\mathrm{A}\}$. As an example, the fifth sequence of S_j is

$$S_5 = \{\mathrm{ABAABABAABABAABA}\} \tag{1}$$

where the ratio of the numbers of the two incommensurate intervals A and B is equal to the golden mean $\tau = (\sqrt{5} + 1)/2$.

As can be seen from expression (1), the sequence possesses symmetric internal structure. For the study of the transmission coefficient, we use the formalism presented in reference [10]. Each layer is characterized by its index of refraction $n_{\mathrm{A\,(B)}}$ and its thickness $d_{\mathrm{A\,(B)}}$. Let matrices T_{AB} and T_{BA} represent the light propagation across interfaces A $\leftarrow$ B and B $\leftarrow$ A. For the case of a normally incident light wave, the matrices are given by

$$T_{\mathrm{AB}} = T_{\mathrm{BA}}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & n_{\mathrm{B}}/n_{\mathrm{A}} \end{bmatrix}. \tag{2}$$

The propagation within one layer can be described by

$$T_{\mathrm{A}} = T_{\mathrm{B}} = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \tag{3}$$

where the phase δ is given by $\delta = 2\pi n_{\mathrm{A}} d_{\mathrm{A}}/\lambda = 2\pi n_{\mathrm{B}} d_{\mathrm{B}}/\lambda$ (λ is the wavelength of the monochromatic electromagnetic wave). Here, we suppose that the index of refraction is wavelength independent and the thicknesses of the optical layers are chosen to give $n_{\mathrm{A}} d_{\mathrm{A}} = n_{\mathrm{B}} d_{\mathrm{B}}$. Therefore, equation (3) is valid only under this condition.

In the following, we will study the optical transmission properties of S_j. For a finite multilayer S_j, which is sandwiched by two media of type A, the corresponding propagation matrix M_j is obtained:

$$M_j = X_{j-1} X_{j-2} Z_{j-2} Z_{j-1} \tag{4}$$

where X_j and Z_j are the propagation matrices of G_j and F_j, respectively. The initial conditions are: $X_0 = Z_0 = T_{\mathrm{AB}} T_{\mathrm{B}} T_{\mathrm{BA}}$, $X_1 = Z_1 = T_{\mathrm{A}}$. Since M_j is a unimodular transfer matrix, the transmission coefficient T_j can be immediately written down:

$$T_j = \frac{4}{m_{11}^2 + m_{12}^2 + m_{21}^2 + m_{22}^2 + 2} \tag{5}$$

where $m_{11}, m_{12}, m_{21}, m_{22}$ are the elements of M_j.

For the case of the ideal Fibonacci sequence (IFS), Kohmoto *et al* [10] predicted the transmission coefficient to exhibit a self-similar behaviour about $\delta = (m + 1/2)\pi$, with $T_{j+3} = T_j$, where m is an integer. The scaling behaviour of the transmission coefficient is characterized by the scale factor

$$f = [1 + 4(1 + J)^2]^{1/2} + 2(1 + J) \qquad \text{where } J = \sin^4\delta[n_A/n_B - n_B/n_A]^2/4.$$

For the SFS (1) studied here, we find that the $\delta = \pi/2$ ($\lambda = \lambda_0$) case also has the very special feature that the matrices M_j satisfy

$$M_{3k} = M_{3k+1} = -M_{3k+2} = -I \qquad k = 0, 1, 2, \ldots \tag{6}$$

where I is the unit matrix. From equation (6), we can see that the matrices have a period of three which differs from the period six of the IFS [10]. This implies that similar scaling behaviour [10] of the transmission coefficient can be observed within a small region around $\delta = (m+1/2)\pi$. Moreover, for any k, the matrix is the identity or the negative identity matrix, which means that perfect transmission can be expected when $\lambda = \lambda_0$.

3. Numerical results

3.1. Properties of optical transmission spectra

In the following numerical investigation, we chose SiO_2 (A) and TiO_2 (B) as two elementary layers, with indices of refraction $n_A = 1.45$ and $n_B = 2.3$, respectively. The optical thickness of each layer is a quarter wavelength ($\lambda_0/4$), where λ_0 is the central wavelength. These conditions imply the phase $\delta = \pi\lambda_0/2\lambda$. For comparison, the transmittances versus λ_0/λ for both IFS and SFS are plotted in figure 1. For the IFS, figures 1(a)–1(c) show the numerical results for F_5 (8 layers), F_6 (13 layers) and F_{10} (89 layers), respectively. In these figures, only one perfect transmission is found around $\lambda = \lambda_0$ (see figure 1(c)); this result can be well explained by the corresponding transfer matrix [18] $M_{10}^F = I$. However, as predicted above, we have unity transmission for any given SFS when $\lambda = \lambda_0$ (refer to figures 1(d)–1(f)). In these three figures, it can also be seen that there are some narrow perfect transmission peaks which are separated by the photonic gaps. Furthermore, on increasing the number of layers in the sequences (16 $\to$ 26 $\to$ 178), more and more transmission peaks with unit transmission coefficient appear. Comparing figure 1(c) with figure 1(f) we can see that for IFS $\longrightarrow$ SFS, although the positions of the transmission peaks are different, the position and the width of the main photonic band gaps remain unchanged. These phenomena in the transmission spectra of the SFS may be useful in the design of optical devices.

To illustrate the scaling property of the transmission spectra of the SFS, we study the transmission coefficients for S_7 (42 layers), S_{10} (178 layers), S_{13} (754 layers) and S_{16} (3194 layers). We enlarge the figures around $\lambda = \lambda_0$ and obtain the four similar pictures shown in figure 2(a) for S_7, figure 2(b) for S_{10}, figure 2(c) for S_{13} and figure 2(d) for S_{16}. The three-cycle feature is well confirmed by these numerical calculations. And the scale factor $f \approx 5.11$, which is the same as that obtained in reference [10]. Note that some resonant peaks of figure 2(a) are much further away from the central position ($\lambda = \lambda_0$) than those of the other three figures. This phenomenon confirms well the theoretical prediction that the three-cycle property of the transmission spectra of S_j can be expected within a narrow region around $\lambda/\lambda_0 = 1.0$. For the given parameters $n_A = 1.45$, $n_B = 2.3$ of the SFS, the corresponding region is [0.95, 1.05].

To show the self-similarity of the transmission spectra about the central wavelength $\lambda = \lambda_0$ (corresponding to the phase $\delta = \pi/2$). We calculate the transmission spectra for S_{17} (5168

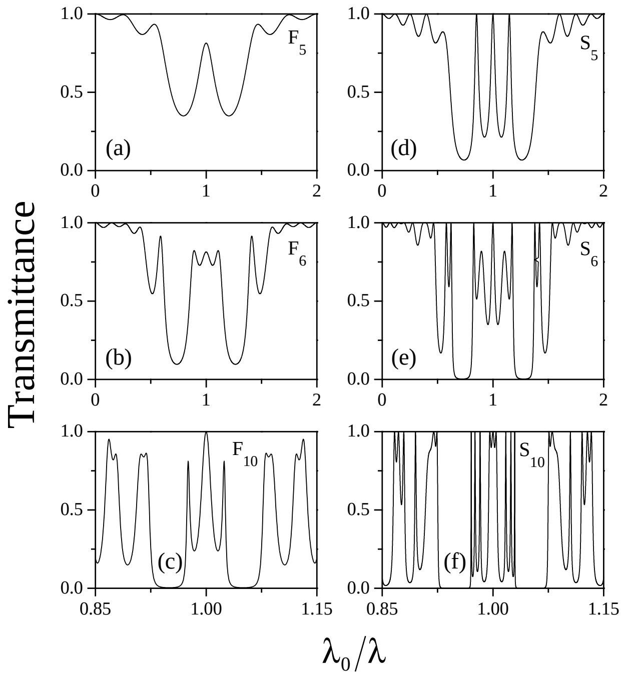

Figure 1. The transmittance versus λ_0/λ for (a) F_5 (8 layers), (b) F_6 (13 layers), (c) F_{10} (89 layers), (d) S_5 (16 layers), (e) S_6 (26 layers), (f) S_{10} (178 layers). The indices of refraction are chosen as $n_A = 1.45$ and $n_B = 2.3$.

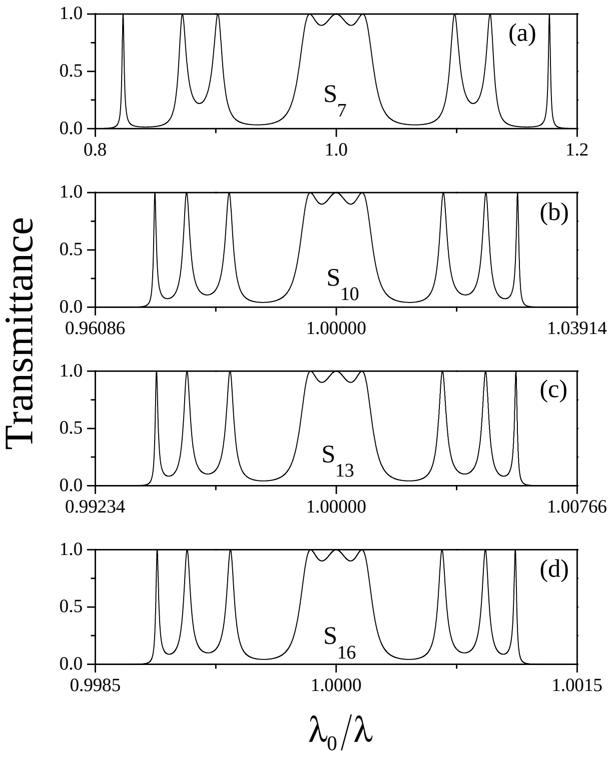

Figure 2. The enlarged figures for (a) S_7 (42 layers), (b) S_{10} (178 layers), (c) S_{13} (754 layers) and (d) S_{16} (3194 layers).

layers). We apply the scale factor $f = 5.11$ to the centre of the optical band repeatedly. And the corresponding enlarged figures are showed in figures 3(a)–3(c). The self-similarity feature is explicitly presented in these figures. Because of the symmetry of the transmission spectra, only half of the numerical results are plotted in these figures. We have employed multifractal analysis ($f(\alpha) \sim \alpha$) [22] and demonstrated that figures 3 do indeed possess multifractal properties.

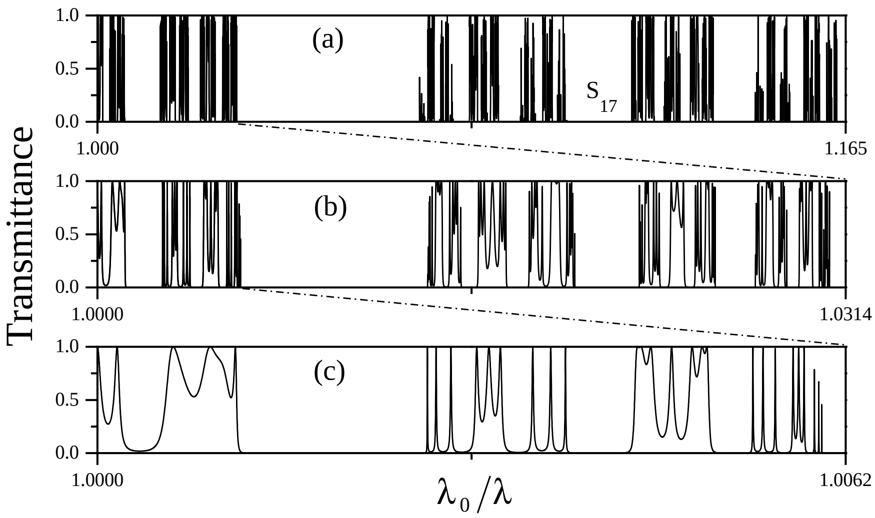

Figure 3. The self-similarity of the transmission spectra around the central wavelength $\lambda = \lambda_0$ for S_{17} is shown. The scaling factor f is equal to 5.11.

3.2. The influence of the randomness

In order to model the optical multilayers more realistically, it is essential to investigate the transmission property of the SFS when the randomness of the film thickness or a systematic error is considered in the system. To explore this aspect of the problem, we suppose that the random thicknesses of the optical layers A (B) are given by

$$d^R_{\mathrm{A\,(B)}}(i) = d_{\mathrm{A\,(B)}}(1 + p\xi_i) \tag{7}$$

where $d_{\mathrm{A\,(B)}}$ is the original thickness of the optical layer, ξ_i is a random number which falls in the range $[-0.5, 0.5]$ and $p = [d^R_{\mathrm{A\,(B)}}(i)_{\max} - d^R_{\mathrm{A\,(B)}}(i)_{\min}]/d_{\mathrm{A\,(B)}}$, which can be used as an indicator representing the randomness of the film thickness.

Figure 4 shows the results of numerical calculations for several situations. Figures 4(a) and 4(b) are obtained for S_6 (26 layers) and S_{10} (178 layers) with 20% randomness ($p = 0.2$). Comparing these two figures with figures 1(e) and 1(f), it is evident that in the case of a very small number of layers (S_6), only a little change in the intensity of the transmission peaks can be observed between figures 1(e) and 4(a). When the number of layers becomes large (S_{10}), great change in the intensity of the transmission peaks can be observed between figures 1(f) and 4(b). Generally speaking, as the number of layers increases, the influence of the randomness will be magnified and all the resonant peaks will eventually disappear. We have studied a series of the transmission spectra of the SFS by increasing the number of layers and by varying the randomness, and found that when the number of layers is less than 110 (S_9) and the

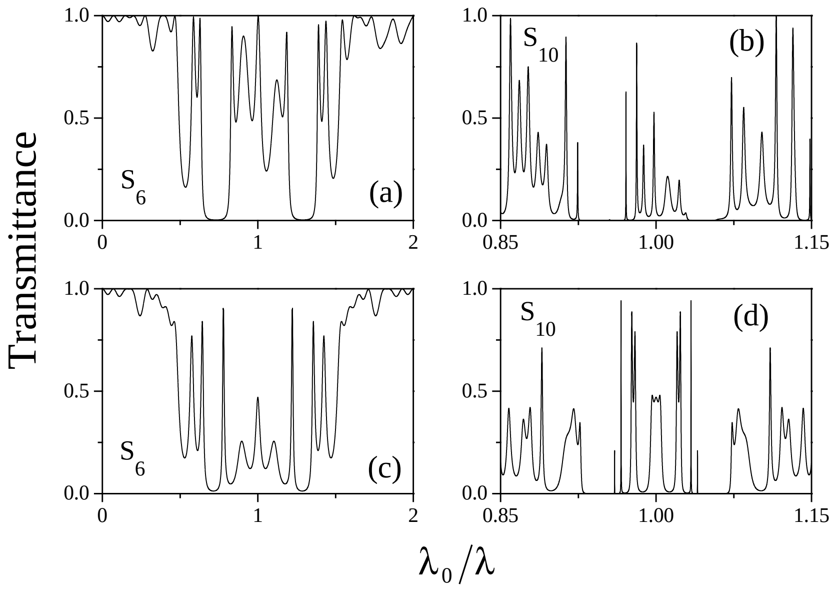

Figure 4. The transmittance for S_6 and S_{10} versus λ_0/λ for several cases. (a) The thickness of the sample is prepared with 20% randomness for S_6. (b) The thickness of the sample is prepared with 20% randomness for S_{10}. (c) The sample is prepared by one misplacement of two neighbouring layers of S_6. (d) The sample is prepared by one misplacement of two neighbouring layers of S_{10}.

randomness is controlled within 5%, the resonant peaks are almost unchanged. Furthermore, one may also find from these figures that although there are substantial changes in the peaks of the transmittances, the photonic gaps are essentially unaffected by the randomness.

In figures 4(c) and 4(d), we show the results for systems with systematic errors (the misplacement of two neighbouring materials), with (c) one misplacement of S_6 and (d) one misplacement of S_{10}. Comparing figures 4(c) and 4(d) with figures 1(e) and 1(f), respectively, we can see that the position and the intensity of the transmission peaks have been changed greatly and the two biggest gaps have been narrowed. Moreover, when the number of misplacements is larger than three, the two biggest gaps of figures 1(e) and 1(f) will disappear and all the quasiperiodic features will be destroyed. Also, from these two figures, we note that the misplacement of neighbouring layers does not change the mirror symmetry of the spectra. This feature can be explained by using equations (2), (3) and (5). Assuming that the two optical phases are $\delta_\pm = \pi/2 \pm \Delta\delta$, respectively, one can easily show that the corresponding transmission coefficients $T_j(\delta_+)$ and $T_j(\delta_-)$ are the same.

4. Summary and discussion

We have studied the light-wave propagation in optical Fibonacci superlattices with symmetric internal structure. We have shown that the combination of aperiodic long-range order and mirror symmetry can greatly enhance the transmission intensity. In particular, for any given index of refraction $n_{A(B)}$, we have obtained a perfect transmission coefficient for the sequences studied at the central wavelength. Around the central wavelength, many sharp perfect transmission peaks have been obtained numerically. And the self-similarity structure of transmission spectra is definitely found. These interesting properties make the SFS a possible candidate design material for multilayered optical filters. And similar results can be expected for other one-dimensional quasiperiodic systems.

It must be noted that, except for the special case of $\lambda = \lambda_0$, we have not provided a satisfactory explanation of the physical nature of the results obtained. In our opinion, to achieve a more comprehensive understanding of these properties, it is important to pay attention to the properties of the transfer matrix. Besides, to pursue experimental evidence of these peculiar transmission properties, further theoretical work is needed.

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